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Electron emission from a semiconductor quantum ring under normally incident radiation

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Abstract

In this paper we investigate the current density due to electrons emitted during normal irradiation of a toroidal quantum ring of a strongly degenerate wide-gap semiconductor. The computed results show that the current density increases in a step-like manner with increase in the incident photon energy. Increased current density can be obtained by reducing the dimensions of the ring. The threshold energy for photoemission becomes an oscillatory function of the cross-sectional radius, and can be used as an important tool for monitoring parameters such as the cross-sectional radius and doping density. The threshold energy is, however, independent of the circumference of the ring.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The quantum nanostructure of semiconductors has become a topic of increasing interest over the past few years, mainly because of their distinctive physical behaviour leading to the invention of many novel devices. In such a semiconductor, one or more dimensions of a semiconductor structure are made comparable to or less than the electron de Bröglie wavelength [1, 2]. Depending on the number of dimensions along which carriers are confined, the semiconductor nanostructures are termed quantum wells (QWs), quantum well wires (QWWs) or quantum boxes or dots (QDs). The electron motion in such structures gets restricted in the direction of reduced dimension, resulting in the quantization of energy states [3, 4]. This quantum size-effect produces significant changes in both the microscopic and macroscopic properties of the semiconductor structure.

The realization of nanoscopic system with wide range of geometries and sizes has been made possible with the rapid progress of fabrication technologies [5]. The particular classes of nanostructures with the ring geometry are being intensively investigated due to the possibility

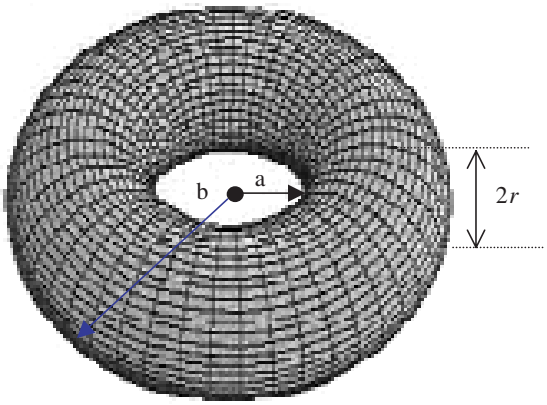

Figure 1. Schematic structure of a toroidal quantum ring.

of experimental observations of the Aharonov–Bohm effect in these structures. Ring-shaped quantum dots with nanometre-scale radii have triggered much interest in theoretical and experimental investigations of their electric and optical properties [6–10]. In many respects, quantum rings are quantum dots with a particular confining potential. The decisive difference in their topology—the 'hole' in their middle—becomes prominent when an external magnetic field is applied. The strong optical transition from valence band to conduction band is the basis for applications of quantum dot (ring) laser emitters, storage devices and fluorescent markers [11–13].

Photo-induced emission is one of the major tools for detailed investigations of the electronic structure of matter and contributes greatly to our understanding of the properties of matter. It is also an important technique that is utilized in various applications, such as determination of energy-band structures and the density of states at the Fermi level, monitoring the dimensions of semiconductor nanostructures, etc [14, 15]. In this paper, the current density due to photoemitted electrons from a toroidal quantum ring of a wide-gap semiconductor has been calculated and computed. The study has been made for normal incidence of photons with energies close to the electron affinity, such that only the conduction band electrons can participate in the photoemission.

The remaining sections of this paper are organized as follows. In section 2 a theoretical background is given. The results and discussion are given in section 3. Finally, in section 4, a summary and conclusions are given.

2. Theoretical background

Let us consider the schematic structure of a toroidal quantum ring as shown in figure 1. This quantum ring can be thought of as a cylindrical quantum wire with its ends connected. Using a cylindrical coordinate system (ρ, θ, z) the time-independent Schrödinger wave equation can be written as

$$-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\theta^2}+\frac{\partial^2\Psi}{\partial z^2}\right]=E\Psi \tag{1}$$

for $0 \leqslant z \leqslant C$ and $\rho \leqslant r$, C and r being the circumference (also the height of the equivalent cylinder) and radius of the cross-section of the ring, respectively, m^* is the electron effective mass and $\hbar$ is Dirac's constant. The electron wavefunction, Ψ, is zero otherwise.

Choosing $\Psi = R(\rho)\Theta(\theta)Z(z)$ and applying the method of separation of variables, equation (1) can be rearranged as

$$-\frac{\hbar^2}{2m^*}\left[\frac{1}{\rho R}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial R}{\partial\rho}\right)+\frac{1}{\rho^2\Theta}\frac{\partial^2\Theta}{\partial\theta^2}\right]-\frac{\hbar^2}{2m^*}\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2}=E \tag{2}$$

for $0 \leqslant z \leqslant C$ and $\rho \leqslant r$.

It can be shown that the wavefunction of an electron in such a quantum ring is given by

$$\Psi \sim \mathrm{e}^{\mathrm{i}2n\pi z/C}\mathrm{e}^{\pm\mathrm{i}m\theta}J_m\left(\frac{\alpha_{ml}}{r}\rho\right), \tag{3}$$

where $n = 1, 2, 3, \ldots$; $m = 0, 1, 2, \ldots$; α_{ml} is the lth zero of the mth-order Bessel function J_m [16] and the energy-eigenvalue becomes

$$E_{nml} = \frac{\hbar^2}{2m^*}\left[\left(\frac{2n\pi}{C}\right)^2+\left(\frac{\alpha_{ml}}{r}\right)^2\right]. \tag{4}$$

Since energy (E) depends on the quantum numbers n, m, l it is denoted by E_{nml}.

The number of states of a quasi zero-dimensional (Q0D) system (for a particular (n, m, l)) per unit volume of the crystal may be defined as

$$N(E) = \frac{g_s g_v}{\pi r^2 C}\delta_{n,m,l} \tag{5}$$

with g_s and g_v being the spin and valley degeneracies, respectively. δ is the Krönecker delta function.

Now, if optical radiation with sufficient energy is incident normally on the plane of the quantum ring of an n-type degenerate semiconductor, electrons from sub-bands in the conduction band are raised to vacuum level. These emitted electrons are free to be collected by electrodes for current. The photoemission current density [17] due to these electrons would be given by the relation

$$J = \frac{1}{2}\sum_{n_{\min}}^{n_{\max}}\sum_{(ml)_{\min}}^{(ml)_{\max}}\alpha_0 q\,\Delta n_0 v_{ml} \tag{6}$$

q being the charge of an electron, α_0 the photon absorption coefficient, v_{ml} the radial component of the velocity of the emitted electrons, assuming that photons are emitted along the radial directions and Δn_0 is the number of electrons per unit volume per sub-band. A factor $1/2$ is introduced to take into account the fact that, statistically, half of the photoexcited electrons are reflected back inside the system. Here $n_{\max}$, $(ml)_{\max}$ are the quantum numbers of the topmost occupied sub-band. $n_{\min}$, $(ml)_{\min}$ are the quantum numbers of the lowermost occupied sub-band, from where emission can occur. Here, $n_{\min}$ is unity and $(ml)_{\min}$ is obtained by the condition

$$h\nu + \frac{\hbar^2}{2m^*}\left(\frac{\alpha_{ml}}{r}\right)^2 \geqslant \chi, \tag{7}$$

χ is the electron affinity and $h\nu$ is the normally incident energy. Depending on the incident photon energy, the emission takes place from all occupied upper sub-bands, the minimum of which corresponds to α_{ml} determined by the above condition. This gives $(ml)_{\min}$.

The velocity of the emitted electron (v_{ml}) is given by

$$v_{ml} = \frac{\hbar}{m^*}\left(\frac{\alpha_{ml}}{r}\right). \tag{8}$$

It may be mentioned here that the incident energy is assumed to be not so large as to cause photoemission directly from the valence band. Some of the incident energy may be used

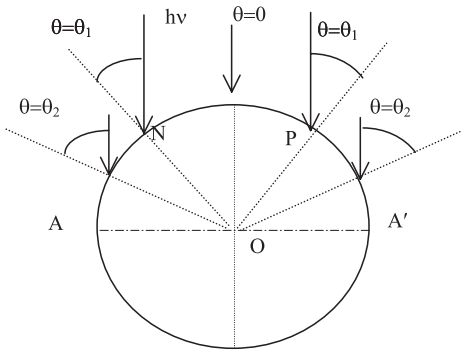

Figure 2. Cross-section of the ring. Light is incident normally on the ring, and thus only the upper half of the ring is irradiated.

up in the absorption process for transition from the valence band to the conduction band, so the photons available for photoemission will be reduced. This can be taken into account by the factor α_0. However, the probability of such absorption is small in the case of a strongly degenerate semiconductor. The ring is grown on a substrate of wide-gap semiconductor [8]. The substrate is usually undoped, so the photoemission from the substrate has also been neglected.

Now let us consider a cross-section of the ring as shown in figure 2. It appears obvious that the incidence of the photon is normal only at the top of the section, i.e. at $\theta = 0$, whereas the photon incidence makes a nonzero angle with the normal, i.e. at $\theta \neq 0$. Let us assume that the incident photon energy is $h\nu$, so that at $\theta = 0$ the normally incident energy is $h\nu$. It can be shown that at an angle θ the component of the incident energy normal to the surface is $h\nu\cos(\theta)$. Thus, the minimum value of the quantum number (ml), i.e. $(ml)_{\min}$ in equation (6), for emission has now become a function of θ also in addition to the incident photon energy. Let us consider that $(ml)_{\min}$ remains unchanged while moving both sides from $\theta = 0$ to θ_1 (figure 2). So, the electrons within the angle NOP will only be responsible for emission from sub-bands with a particular $(ml)_{\min}$. This is the lowermost value of $(ml)_{\min}$ denoted by $(ml)_{\min 1}$. If θ increases further ($>\theta_1$), the emission from $(ml)_{\min 1}$ is not possible (equation (7)). So, $(ml)_{\min} = (ml)_{\min 2} > (ml)_{\min 1}$. As discussed above, there exists $\theta = \theta_2$, up to which $(ml)_{\min}$ remains unchanged, i.e. $(ml)_{\min} = (ml)_{\min 2}$.

Assuming uniform angular distribution of electrons, the number of such electrons per unit volume corresponding to $(ml)_{\min} = (ml)_{\min 2}$ is $(2\theta_1\Delta n_0/\pi)$. Similarly, the number can be calculated for other $(ml)_{\min}$. Thus, the expression for J can be rewritten in the form

$$J = \frac{1}{2}\sum_{n=1}^{n_{\max}}\left[\sum_{(ml)_{\min 1}}^{(ml)_{\max}}\frac{2\theta_1}{\pi}\alpha_0 q\,\Delta n_0 v_{ml} + \sum_{(ml)_{\min 2}}^{(ml)_{\max}}\frac{2\,(\theta_1-\theta_2)}{\pi}\alpha_0 q\,\Delta n_0 v_{ml} + \cdots\right]. \tag{9}$$

It is obvious from figure 2 that there will be no photons incident below the axis line AA'. The electron density Δn_0 per quantized level can be obtained by multiplying equation (5) with the Fermi–Dirac distribution function. Thus, the expression for the total number of electrons per unit volume per quantized level can be written as

$$\Delta n_0 = \frac{g_s g_v}{\pi r^2 C}\frac{1}{1+\exp\{(E_{nml}-E_F)/(k_B T)\}}, \tag{10}$$

E_F being the Fermi energy measured from the edge of the conduction band, k_B the Boltzmann constant and T the temperature.

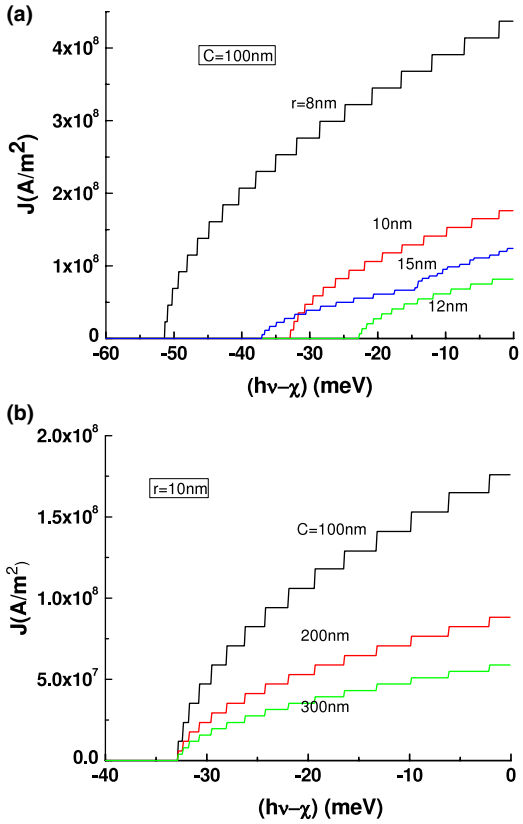

Figure 3. Photoemission current density (J) as a function of ($h\nu - \chi$) (a) for different values of its radius of cross-section with ring circumference (C) of 100 nm and, and (b) for different values of ring circumference with cross-sectional radius of 10 nm taking n-GaAs as an example at a temperature of 4.2 K.

Thus, the photoemission current density due to the electrons emitted from the surface of a quantum ring of circular cross-section of a wide gap semiconductor is given by

$$J = \frac{g_s g_v \alpha_0 q \hbar}{2\pi r^3 C m^*} \sum_{n=1}^{n_{\max}} \Bigg[\sum_{(ml)_{\min 1}}^{(ml)_{\max}} \frac{2\theta_1}{\pi} \frac{\alpha_{ml}}{1 + \exp\{(E_{nml} - E_F)/(k_B T)\}} + \sum_{(ml)_{\min 2}}^{(ml)_{\max}} \frac{2(\theta_1 - \theta_2)}{\pi} \frac{\alpha_{ml}}{1 + \exp\{(E_{nml} - E_F)/(k_B T)\}} + \cdots \Bigg]. \tag{11}$$

The maximum values of $n_{\max}$ and $(ml)_{\max}$ are determined by the assumed fact that there is no occupancy above the Fermi level since the semiconductor is a highly degenerate one.

3. Results and discussion

Using equation (11) and assuming $\alpha_0 = 1$, $g_s = 2$, $g_v = 1$, the photoemission current density has been computed for a strongly degenerate n-GaAs ($m^* = 0.067 m_0$) at 4.2 K neglecting the formation of band-tails, if any. The electron affinity (χ) of GaAs is 4.07 eV. However, in the

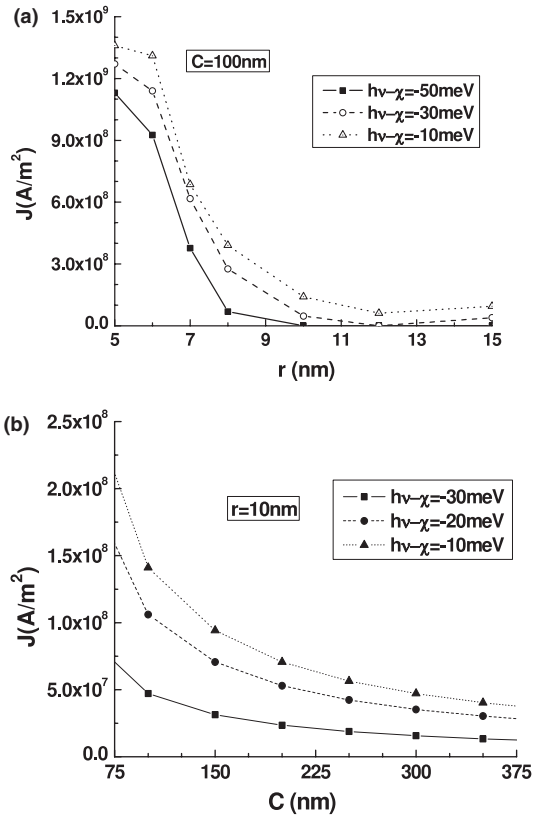

Figure 4. Photoemission current density (J) as a function of the dimension of the quantum ring for different values of ($h\nu - \chi$). (a) Variation with radius (r) of the cross-section of the wire for a ring circumference (C) of 100 nm. (b) Variation with C for $r = 10$ nm are shown.

present analysis, it is the difference ($h\nu - \chi$) which has significance and so computations are done considering ($h\nu - \chi$) instead of incident energy ($h\nu$) as a variable. The photoemission current density as a function of ($h\nu - \chi$) is shown in figure 3. In figure 3(a) cross-sectional radius has been taken as a parameter keeping the circumference constant, while in figure 3(b) circumference has been taken as a parameter keeping the cross-sectional radius constant. In general, the current density varies in a step-like manner with the incident photon energy. The step width increases with increase in photon energy and the step height decreases with increase in dimensions (r or C). In figure 3(a) the threshold energy for which the electron starts emitting becomes a function of cross-sectional radius (r), while it is independent of the circumference (C) of the ring. This is due to the fact that the velocity of electron emission depends only on the radial quantum numbers, and not on the quantum numbers arising from circumferential motion (see equation (7)). The variation of threshold energy with r will be shown in detail in a separate graph later.

The current density as a function of r is shown in figure 4(a). As r is increased in small steps, the current density changes in a non-monotonic manner. It first decreases rapidly, attains a minimum and then increases slowly. But this slow and gradual increase can be observed only if $h\nu$ is kept fixed at a high value. This is due to the reasons given in the previous paragraph. The variation of current density with C is shown in figure 4(b), where the current density decreases monotonically. In this case, the current density gradually decreases with the circumference and ultimately, for very large values of C, it becomes the current density in a quantum well wire. So, the reduced dimension of the ring gives rise to a larger emission current density.

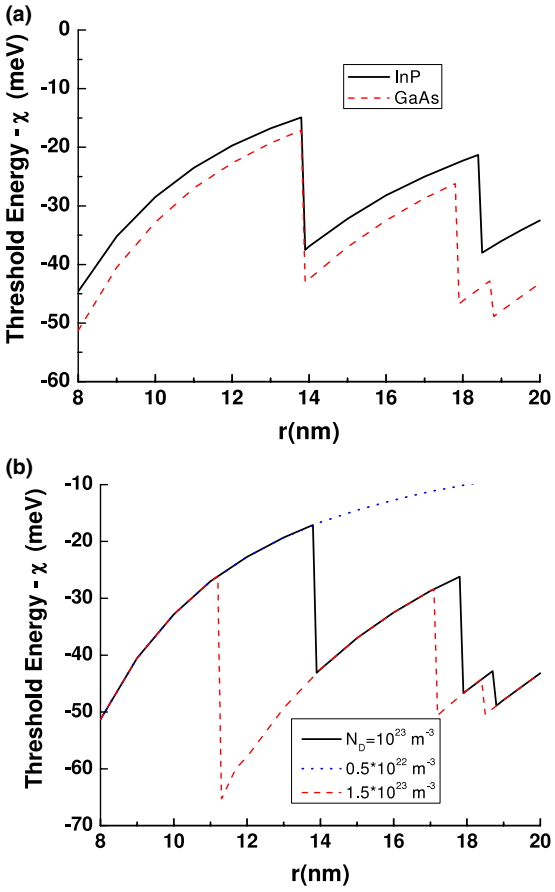

Figure 5. Threshold energy (with χ subtracted) of photoemission from a semiconductor quantum ring at 4.2 K as a function of radius (r). (a) For two different materials, n-GaAs and n-InP, using a doping density of 10^{23} m^{-3}. (b) For three different doping densities of a degenerate n-GaAs: 0.5×10^{22} m^{-3}, 10^{23} m^{-3} and 1.5×10^{23} m^{-3}.

The variation of threshold photon energy (measured with respect to χ) with radius of the cross-section of the ring is shown in figure 5. This shows that the threshold energy increases gradually with increase in $h\nu$, but decreases abruptly at some values of r. The same thing is repeated as the radius is increased. This oscillatory nature can be explained as follows. When r increases, the energy levels are lowered and hence the threshold energy increases. But, after a particular value of r, a higher energy level comes below the free-space energy and the threshold energy for emission drops abruptly to a minimum. Thus, the threshold energy is an oscillating function of the radius of circumference; however, the oscillation period is non-uniform, determined by the radial quantum number. In figure 5(a), a comparison is shown for two different materials: GaAs and InP as an example, for a fixed doping density. The sub-band spacing is smaller in InP than in GaAs due to the larger electron effective mass in the former, and so the threshold energy is higher in InP at a particular r. In figure 5(b), the plot is shown for the same material but with a different doping density. With increased doping density, the occupancy of higher sub-bands increases, and so the drop in the threshold energy occurs at lower values of r. The period of oscillation also increases with increase in doping density. Thus, the threshold energy plot as a function of r can be used as a tool to estimate the cross-sectional radius of the quantum ring. For a known cross-sectional radius, the calibrated plot can also be utilized to control the doping density of the degenerate sample.

4. Summary and conclusion

The photoemission current density from a toroidal quantum ring of a strongly degenerate semiconductor has been calculated. The emission is shown to be step-like in nature with the incident photon energy. As the emission is along the radial directions, the threshold photon energy for emission becomes strongly dependent on the radius of cross-section of the ring, and by the same reasoning it is independent of the circumference of the ring. Thus, the threshold energy plot can be used for monitoring the cross-sectional radius of the ring, the doping density of the degenerate semiconductor, etc. The photoemission current density gives an idea of the quantized energy states inside the ring, and with a suitable choice of materials and small dimensions, efficient optical emitters and detectors in certain wavelength ranges can be devised.

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